Variational Autoencoder for Turbulence Generation

Kevin Grogan
Stanford University

Introduction

Turbulence

- The chaotic motion of fluids
- Applications
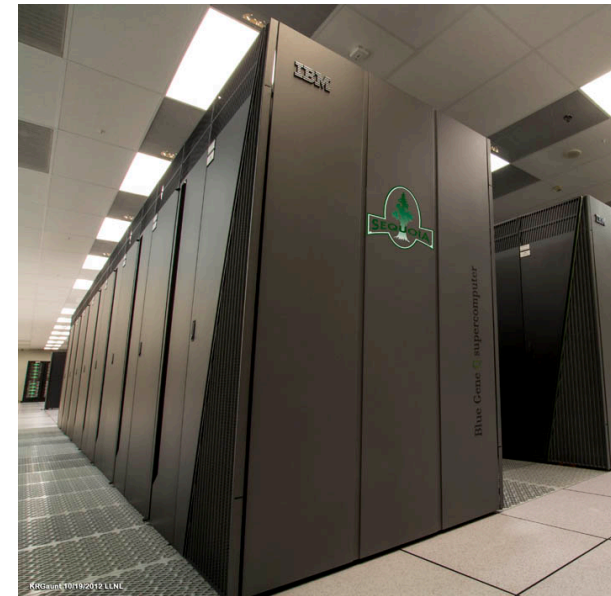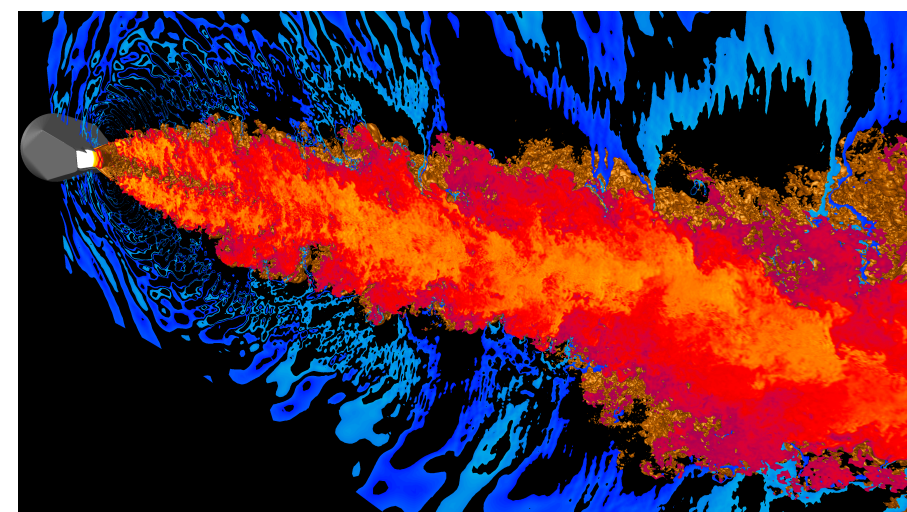 - Airframe design
  - Turbomachinery
  - Combustion

Navier Stokes Equations

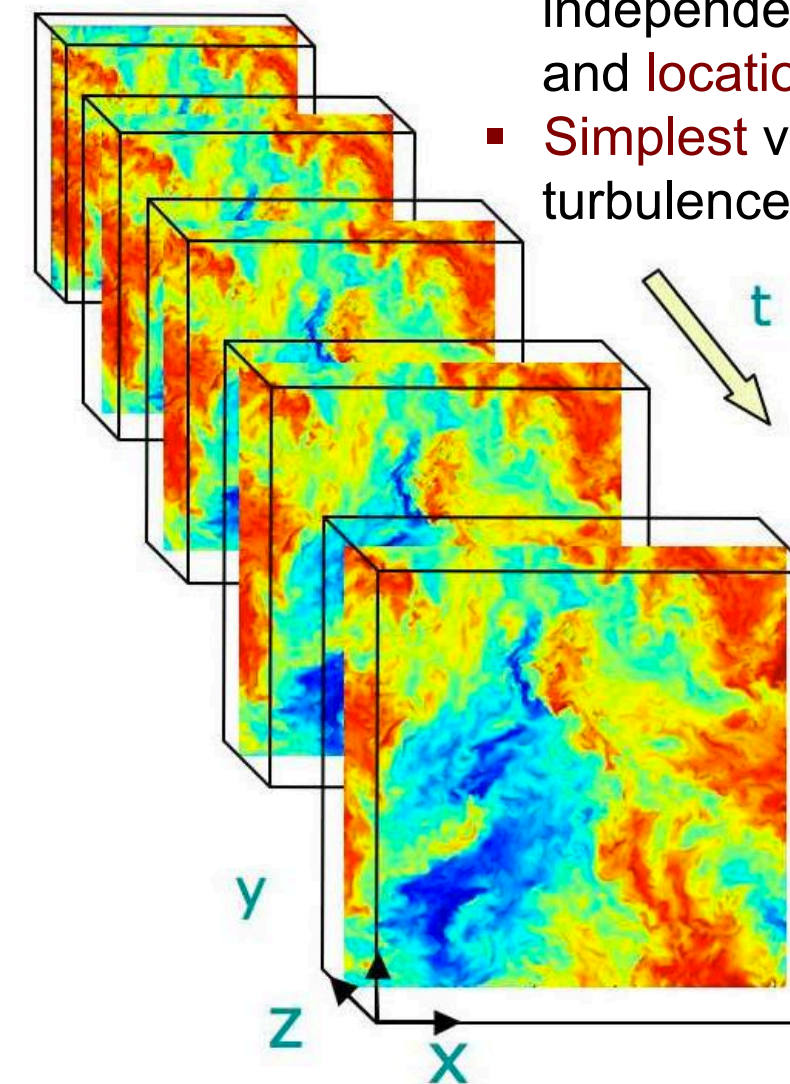$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla\left(\frac{p}{\rho}\right) + \nu\nabla^2 u$$

$$\nabla \cdot u = 0$$

- Consists of a force balance and mass conservation
- Famously complex (Millenium Prize)
- Computations of highly turbulent flows are intractible.

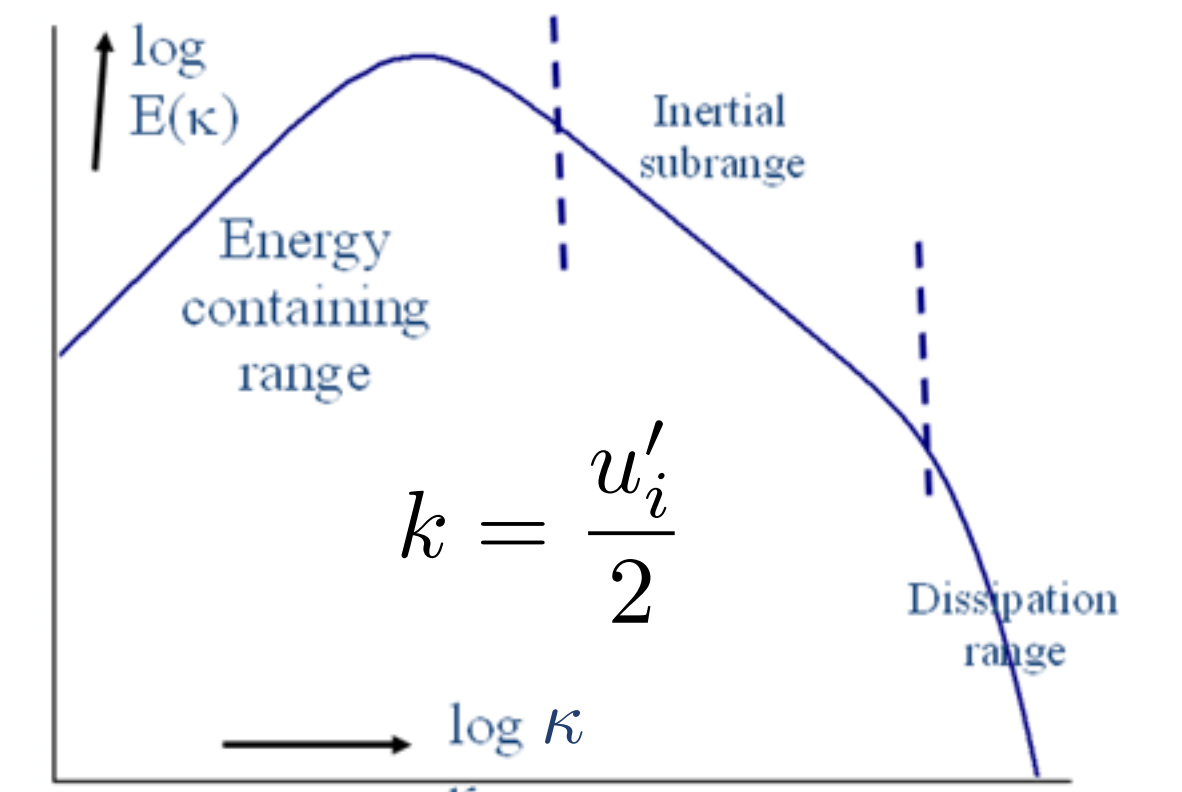### Homogeneous Isotropic Turbulence (JHTDB)

- Turbulence independent and local
- Simple turbulence
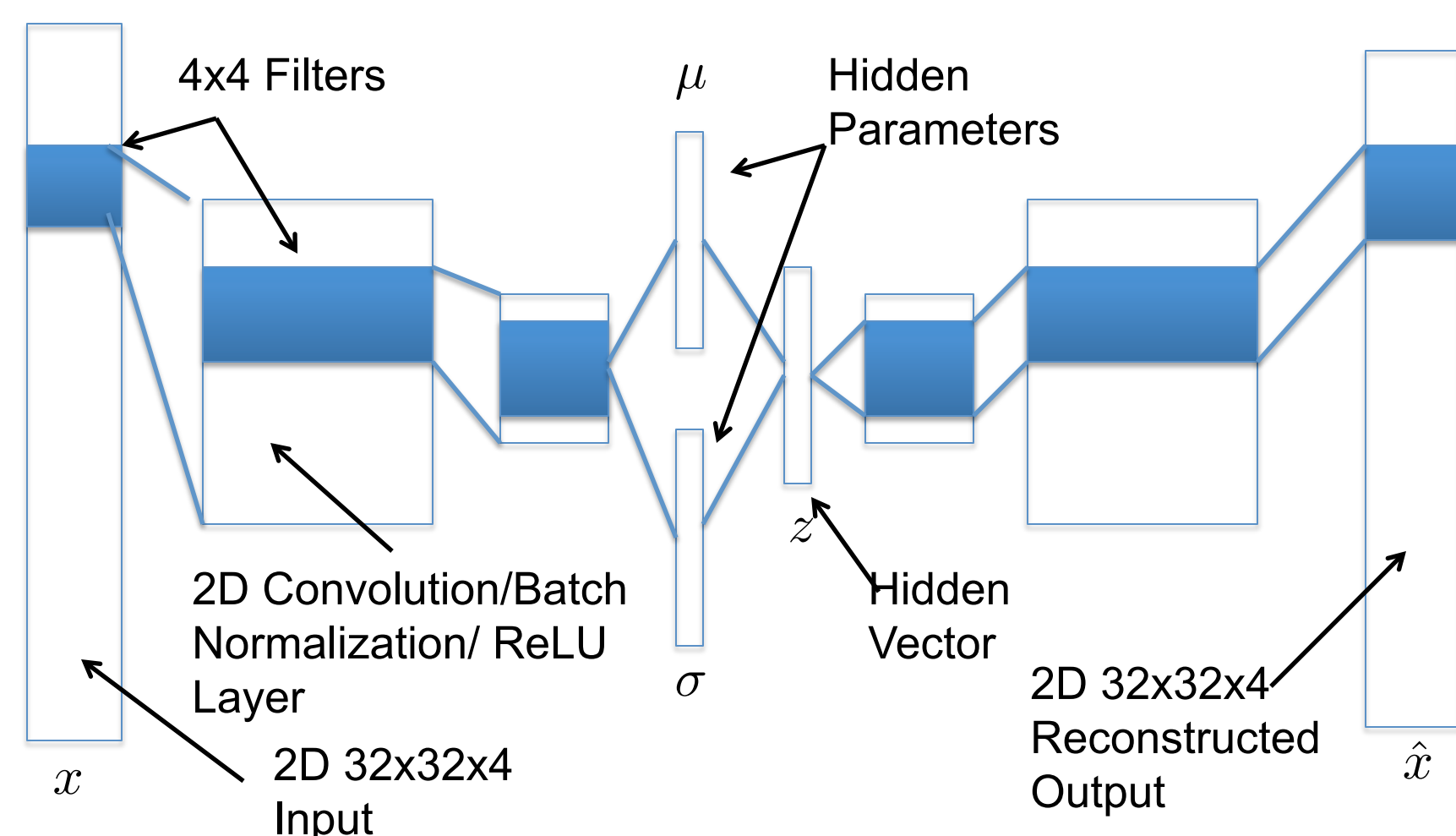

Turbulent Kinetic Energy Spectrum

$$k = \frac{u'_i}{2}$$

- Turbulence is local
- Deep convolutional networks may provide a sound description of turbulent structures.
- A generative model may be able to produce turbulence flow for modeling purposes.

Model

Convolutional Variational Autoencoder

- Utilizes an encoder and decoder network
- Encodes the parameters of a hidden distribution
- Decodes a lower dimensional sample of hidden variables
- Minimizes a lower bound on the likelihood function

4x4 Filters

μ

Hidden Parameters

2D Convolution/Batch Normalization/ ReLU Layer

z

Hidden Vector

σ

2D 32x32x4 Input

2D 32x32x4 Reconstructed Output

x

$\hat{x}$

$$\mathcal{L}_i = \mathbb{E}_z[\log p_\theta(x^{(i)}|z)] - D_{\mathrm{KL}}(q_\phi(z|x^{(i)})||p_\theta(z))$$
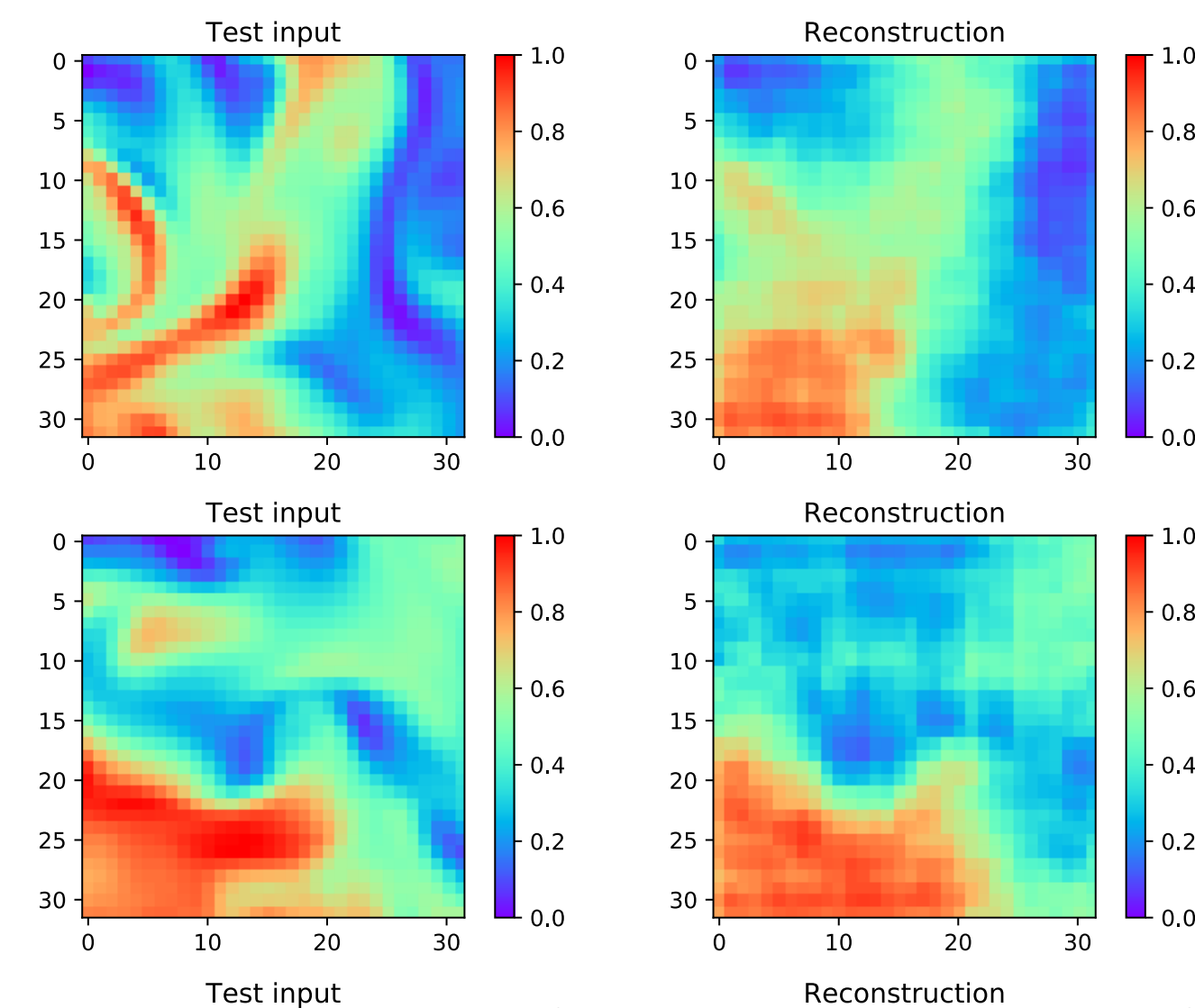
Loss per the example

Expected reconstruction by decoder network

Distribution approximated by encoder network

Prior distribution

Results

Reconstruction Input

Summary and Future Work

- Simulation of turbulent flow is a computationally expensive task
- A generative model could help alleviated the burden of calculating these flows.
- A variational autoencoder is used to learn the attributes of a turbulent flow from a database.
- Qualitative agreement was found between the reconstruction and the input.
- Future work (for this project and beyond) includes:
  - Evaluating the error loss on the test set
  - Examining the generated data
  - Extending the model to a 3D ConvNet
  - Incorporating different flows.